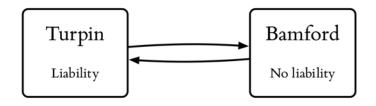


# RENVOI AND THE BARBER

### James Grimmelmann

ANIEL O'HIGGINS IS SHAVING with a scalpel made by the Sweeney Surgical Company when the blade breaks, gashing his neck. O'Higgins lives in the state of Turpin and Sweeney is incorporated there, but the injury takes place during a business trip to the neighboring state of Bamford. Misuse of this sort is a defense to a products liability claim under Bamford tort law, but not under Turpin tort law. Turpin choice-of-law rules select the place of injury; Bamford choice-of-law rules select the parties' common domicile. O'Higgins sues in a Turpin state court. Which law should the court apply?

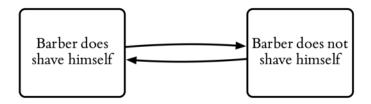
The most amusing answer goes like this: We start with the forum's choice-of-law rules. Here that is Turpin, where O'Higgins filed suit. Turpin choice of law selects Bamford, so we look to Bamford law. Bamford choice of law now selects Turpin, so we return to Turpin law. Turpin selects Bamford again, Bamford selects Turpin back again, and around and around it goes.



James Grimmelmann is a professor at Cornell Tech and Cornell Law School. Copyright 2019 James Grimmelmann. He offers this article to the rest of the world under Creative Commons Attribution 4.0 International license, creativecommons.org/licenses/by/4.0.

This is the paradox of *renvoi*. If state A's law applies, then state B's law applies. But if state B's law applies, then state A's law applies. Both possibilities logically lead to each other. Paradox!

Scholars and teachers like to connect the *renvoi* paradox to another famous paradox: Bertrand Russell's Barber Paradox. In a certain town there is a barber, who shaves all the men who do not shave themselves. Does the barber shave himself? If so, then it follows that he is not in need of the barber's services, so he is not shaved by the barber, i.e., he does not shave himself. But if the barber does not shave himself, it follows that he does need the barber's services, so he is shaved by the barber, i.e., he does shave himself. Both possibilities logically lead to each other. Paradox!



This is as far as the discussion usually goes. The paradox of *renvoi*'s infinite regress is more of a theoretical problem than a practical one. There are no reports of cases in which a judge starved to death while stuck in an infinite *renvoi* loop. Russell presented the Barber Paradox to illuminate some problems in the foundations of mathematical logic and set theory, which is about as far from choice of law as one can get.<sup>2</sup> So while the Barber Paradox is amusing, it is not thought to have much to say about *renvoi*.

I think this is wrong, because it misses an important part of the story: how mathematicians escape the Barber Paradox. It is fundamentally a paradox of self-reference: the barber who shaves himself.<sup>3</sup> *Renvoi* too is a

<sup>&</sup>lt;sup>1</sup> See, e.g., David Alexander Hughes, The Insolubility of Renvoi and Its Consequences, 6 J. Priv. Int'l L. 195, 216-17 (2010); Laurence Goldstein, Four Alleged Paradoxes in Legal Reasoning, 38 Cambridge L.J. 373, 380-82 (1979); J.C. Hicks, The Liar Paradox in Legal Reasoning, 29 Cambridge L.J. 275, 284-86 (1971).

<sup>&</sup>lt;sup>2</sup> For a careful and accessible introduction to "naïve" set theory, its notation, and its underappreciated relevance to legal reasoning, see Jeremy N. Sheff, *Legal Sets*, Cardozo L. Rev. (forthcoming), ssrn.com/abstract=2830918.

<sup>&</sup>lt;sup>3</sup> The other infamous paradox of self-reference is the Liar Paradox. Consider the sentence "This sentence is false." Is it true or false? Either answer implies the other. See generally

problem of self-reference: ordinary choice of law blows up into paradox not when one state's laws refer to another's, but when a single state's laws refer back to themselves. Set theorists responded to the Barber Paradox by modifying their theories to exclude the kind of self-reference that can go so badly wrong, and this is the purpose of *renvoi* rules.

#### **MATHEMATICS**

The Barber Paradox is Russell's playful restatement of a serious flaw he identified in Gottlob Frege's attempt to put logic and mathematics on a philosophically rigorous footing. In trying to formalize the intuitive idea that two functions are the same if they have the same values, Frege implicitly relied on what would today be called the "unrestricted Axiom of Comprehension": given a property (e.g., "is green") there is a set of all things having that property. Slightly more formally, suppose P(x) is a predicate: a logical formula that is true if x has the given property and is false if x does not have it. For example, if P(x) is the predicate "x is green" then P(Kermit the Frog) is true and P(Miss Piggy) is false. The unrestricted Axiom of Comprehension states: given any predicate P(x) there is a set S of the objects x for which P(x) is true. In the universe consisting of the two objects Kermit the Frog and Miss Piggy, the set S corresponding to the predicate "x is green" is {Kermit the Frog}. Kermit is in S because he is green; Miss Piggy is not in S because she is not green.

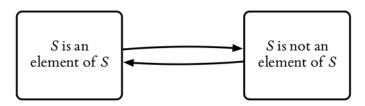
Unfortunately, the unrestricted Axiom of Comprehension leads directly to paradox. In a letter to Frege, Russell observed that if P is the predicate "x is not an element of x", 5 paradox immediately results. Feed P into the

Jon Barwise and John Etchemendy, The Liar: An Essay on Truth and Circularity (1989). Despite their obvious kinship, the Barber Paradox and Liar Paradox raise different issues — most obviously, the Liar Paradox explicitly appeals to a conception of truth — and I will focus on the Barber Paradox.

<sup>&</sup>lt;sup>4</sup> Although he used the Barber Paradox as a metaphor to illustrate his set-theoretic paradox, Russell denied that he had originated this formulation of it. *See* Bertrand Russell, *The Philosophy of Logical Atomism*, 29 Monist 345, 354-55 (1909).

<sup>&</sup>lt;sup>5</sup> Actually, he wrote, "Let w be the predicate: to be a predicate that cannot be predicated of itself." Letter from Bertrand Russell to Gottlob Frege (June 16, 1902), in From Frege to Gödel: A Source Book in Mathematical Logic 124, 124-25 (Jean van Heijenoort ed. 1967). But it is a little clearer to translate this into the language of sets.

Axiom of Comprehension and let *S* be the resulting set. Now ask if *S* is an element of itself. Suppose it is. Since *S* is an element of *S*, by the definition of *S* the predicate *P* is true of it, and thus by the definition of *P* it follows that *S* is not an element of *S*. But if *S* is not an element of *S*, then by definition of *P* it follows that *P* is true of *S*, and then by the definition of *S* it follows that *S* is a member of *S*. Either assumption leads to its opposite.



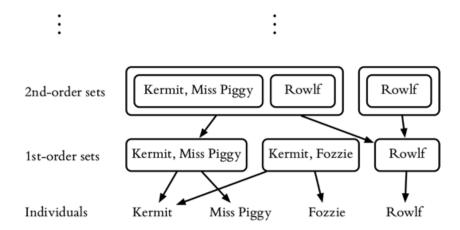
The Barber Paradox is Russell's Paradox dressed up with an apron and hot towels. The barber is the set *S*, he shaves anyone who is a member of *S*, and the paradoxical predicate is "does not shave himself." It is in this form that it captured the imaginations of mathematicians and conflict-of-laws scholars. Any assumption — that *S* contains itself or does not, that the barber shaves himself or does not, that the law of state X applies or does not — apparently leads to a contradiction.

Russell's Paradox arises because the unrestricted Axiom of Comprehension is too strong: by asserting that there is a set *S* corresponding to every predicate *P*, it immediately thrusts *S* into the domain of objects to which *P* could apply. This is the source of the self-reference Russell exploited with his paradoxical predicate.

The story does not stop there. Mathematics has not ground to a halt. Russell himself demonstrated one way to avoid the paradox. In their *Principia Mathematica*, Russell and Alfred North Whitehead prevented sets from being elements of themselves by classifying sets into an infinite hierarchy of "types." At the very bottom are "individuals": things like barbers and bumblebees which are not sets and do not contain anything. At the first level are "first-order" sets whose elements are objects: the set containing the numbers 23 and 42, the set of all bumblebees, the set of barbers named "Fred," and so

<sup>&</sup>lt;sup>6</sup> Alfred North Whitehead & Bertrand Russell, Principia Mathematica (1910-13). Russell laid out the theory of types in Bertrand Russell, *Mathematical Logic as Based on the Theory of Types*, 30 Am. J. Math. 222 (1908), *reprinted in* From Frege to Gödel, *supra* note 5, at 150.

on. At the second level are "second-order" sets each of whose elements is a first-order set. For example, Kermit the Frog and Miss Piggy are objects. {Kermit, Miss Piggy} is a first-order set, and so is {Rowlf}. At the next level, {{Kermit, Miss Piggy}, {Rowlf}} is a second-order set. The elements of an nth-order set are n-1th-order sets, with no exceptions. In this diagram, the arrows from a set to its elements can only go down exactly one level, never sideways or upwards.



Thus, the *Principia* is able to use a restricted version of the Axiom of Comprehension. Given any predicate P(x) expressing a property of n-1th-order sets, there is an nth-order set S of the objects x for which P(x) is true. This version works because it refuses to mix types. Predicates only apply to objects at specific levels in the type hierarchy. Sets only contain objects one level lower. The paradoxical predicate P(x) cannot even be expressed, because "x is not a member of x" mismatches the types: x is at the same level as itself, whereas "is a member of" holds only between objects at level n-1 and at level n.

Russell and Whitehead's type system turned out to be too cumbersome for most applications. But it solves the problem of self-reference. It does so by imposing a hierarchy: objects at a specific level in the hierarchy can never refer to themselves or objects at a higher level, only to objects at a lower level. Other approaches to the foundations of mathematics use different hierarchies. For example, consider the leading formalism relied on

by mathematicians today to capture what sets are and how they work, known as "ZF" set theory after its creators, Ernst Zermelo and Abraham Fraenkel. ZF set theory describes the universe of sets in stages. At each stage, it describes new sets in terms of sets that have already been described. This approach automatically precludes defining sets in terms of themselves: they aren't available, since they haven't been described yet. Instead of the unrestricted Axiom of Comprehension, Zermelo-Fraenkel set theory uses a narrower but safer version. Given any predicate P(x) and an already-defined set T, there is a set S of the objects X which are elements of T and for which P(X) is true. This too solves the problem of self-reference. Consider Russell's paradoxical predicate P(X). To use it to define a set S, we must first start with some other set T. Pick any element X in T. Is X a member of itself? No it is not. Thus Y(X) is not true, so X is not an element of S, either. Repeat for all the other members of T. None of them are in S. So S is a well-defined set, and it is empty. There is no paradox.

This gives another perspective on what goes wrong in the Barber Paradox. It is perfectly meaningful to talk about barbers, about the set of people a barber shaves, and about barbers who do and do not shave themselves. The problem comes when we describe a barber who supposedly shaves all the men who do not shave themselves *and use this description to claim that such a barber exists*. There can be no such barber, not if the universe of men he might or might not shave includes himself.<sup>8</sup>

As soon as we stop trying to act as though there is such a barber, the paradox melts away. The easiest way to block the paradoxical definition is

<sup>&</sup>lt;sup>7</sup> Axiomatic set theory is not for the mathematically timid. The ideas are simple enough at first, but they must be stated with obsessive precision. A relatively accessible introduction is Herbert Enderton, The Elements of Set Theory (1997). The reader seeking more insight into how axiomatic set theory avoids paradox will learn much from Keith Devlin, The Joy of Sets (2nd ed. 1994).

 $<sup>^{\</sup>rm 8}$  Computer scientist Edsger Dijkstra put the point with characteristic pithiness:

For the barber of the village we have [an equation defining the barber] and that equation has no solution. Conclusion: the village has no barber. Where is the paradox?

Probably I am very naive, but I also think I prefer to remain so, at least for the time being and perhaps for the rest of my life.

Edsger W. Dijkstra, EWD923A: Where is Russell's "Paradox"? (1985), www.cs.utexas.edu/users/EWD/ewd09xx/EWD923a.PDF.

to prevent barbers from being defined in terms of themselves. Exclude the barber from the universe of men he might shave, and the self-referential cycle never arises.

Different mathematical formulations of set theory exclude the "barber" in different ways, but they all exclude the barber. The "NBG" version of axiomatic set theory (named after John von Neumann, Paul Bernays, and Kurt Gödel) uses a version of the Axiom of Comprehension in which objects called "classes" can be defined in terms of arbitrary predicates, but not all classes are sets. To put things a little loosely, a female barber (class) can shave (contain) all men (sets) who do not shave (contain) themselves, since the barber (class) is not a man (set). More recently, "non-well-founded" set theory allows for sets that *contain* themselves, directly or indirectly, but the sets are *defined* in terms of non-self-referential objects (directed graphs). These sets can be circular, but they are not paradoxical: each set either contains itself or doesn't. In all of these systems — Russell-Whitehead set theory, ZF set theory, NBG set theory, and non-well-founded set theory — there are no definitions based on true self-reference, only definitions in terms of simpler objects.

### LAW

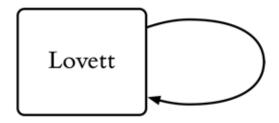
Renvoi is paradoxical in the sense of the Barber Paradox: it is a problem of self-reference run amok. 10 The paradox doesn't arise when two states have different substantive laws. It doesn't arise when two states' choice-of-law rules select each other. It doesn't even arise when states have different substantive laws and they have choice-of-law rules that select each other. It arises only when both of these things are true and there is

For an accessible (relatively speaking) introduction to NBG and non-well-founded set theory, see generally Devlin, supra note 7.

<sup>&</sup>lt;sup>10</sup> Self-reference was arguably *the* defining problem of logic in the twentieth century, with major implications for mathematics, computer science, and philosophy. Paradoxical self-reference in set theory can be overcome with careful definitions, but in the 1930s, Kurt Gödel, Alan Turing, and Alfred Tarski used self-referential paradoxes to demonstrate the existence of inherent limits on deduction, computation, and logical truth, respectively. *See generally* Peter Smith, An Introduction to Gödel's Theorems (2nd ed. 2013). The canonical edition of the canonical popular book on self-reference in logic and beyond is Douglas R. Hofstadter, Gödel, Escher, Bach: An Eternal Golden Braid (20th Anniv. Ed. 1999).

added the mistaken axiom that to "select" a state's law means to adopt every last iota of it.

In fact, treating a choice-of-law selection as selecting whole law leads straight to paradoxical *renvoi* self-reference even if only one state is involved. Suppose that the state of Lovett selects its own law in battery cases. A plaintiff who is injured in Lovett sues in Lovett. The forum court starts by consulting Lovett's choice of law rules, which direct it to apply Lovett law. So the court consults Lovett's choice of law rules, which direct it to apply Lovett law. So the court consults Lovett's choice of law rules, which direct it to apply Lovett law . . . *ad infinitium*.



Obviously this is absurd, because in the real world when a state's choice of law rules direct it to apply its own law, its courts then proceed to apply its domestic law rather than its whole law. In Russell-Whitehead terms, "domestic law" and "choice of law" are at different levels of a type hierarchy: domestic law is a zeroth-order substantive rule, but whole law can include higher-order choice-of-law rules. To say that a state's choice-of-law rules select its own "law" is just to say that its higher-order choice-of-law rules select its own zeroth-order substantive rule. This point is obvious when a state's choice of law rules point to its own law — so obvious that no one even contemplates the alternative possibility that the "law" they select could be whole law rather than domestic law.

There is no paradox here about two competing and incompatible rules. Instead, the theoretical paradox arises from the attempt to select the exact same law doing the selecting. The problem is the self-reference, not the dueling choices. The problem with Frege's set theory was sets defined in terms of themselves: Russell's self-contradicting predicate was just the example that laid bare the problem.

The same is true in the multi-state context. There is nothing logically fishy about a choice-of-law rule that takes other states' law into account.

Nor is there anything suspect about following another state's choice-of-law rules back to the first state's law, directly or indirectly. Paradoxical self-reference only arises if one set of laws refers back to the exact same set of laws. In the simplest case, this would occur if the forum court refers a second time to the totality of its own state's laws.

Paradox is averted if whenever a court consults one state's laws a second time, it consults *a subset of the set it consulted the first time*. If Turpin choice of law selects Bamford and Bamford choice of law selects Turpin, everything is fine as long as the second examination of Turpin law involves something less than Turpin's whole law. It is not necessary that the process arrive back at Turpin's domestic law: a state can have more than two levels to its choice-of-law hierarchy. If there are only a finite number of levels and the process descends a level each time it bounces back, the process will eventually bottom out. The descent could happen because Turpin's own *n*th-level law directs the court to examine only lower-level laws when it arrives back, or because one of the other legal systems it has passed through cares only about lower-level Turpin laws. Either way, there is no self-reference when the cycle arrives back at Turpin.

The logical necessity of cycle-breaking does not tell us anything interesting about how or when to break cycles. It provides us no useful information on which aspects of its own or another state's law a state should tell courts to ignore as they descend from one choice-of-law level to a lower one. It just says that the courts must ignore something. Kermit Roosevelt is right to say, "Moreover, focusing on the logical aspect of the problem detaches it from the context of choice of law and obscures the extent to which *legal* analysis can tell us something about which solutions are plausible and which are not."

Still, I think "focusing on the logical aspect of the problem" can be helpful in that it helps us understand exactly what a court is doing when it follows one *renvoi* rule or another. It is *ignoring part of a state's law*. The important decision involved in adopting a *renvoi* rule is *which part to ignore*. No state can ever truly have a choice of law rule that follows the whole of

<sup>&</sup>lt;sup>11</sup> Actually, the process also bottoms out if there are an infinite number of levels, as long as the number is an ordinal. But this is not the time or place to get into transfinite recursion.

<sup>&</sup>lt;sup>12</sup> Kermit Roosevelt III, Resolving Renvoi: The Bewitchment of Our Intelligence by Means of Language, 80 Notre Dame L. Rev. 1821, 1837 (2005).

another state's law. It can claim to have such a rule, but only so long as it is never truly put to the test by confronting another state which also claims to have a similar rule. If two such states really do face off, the forum court will end up saying, in effect, that one of the two states didn't really mean it. It is cruel and unhelpful to put courts in this position, because it forces them to justify their *renvoi* decisions on the basis of legal fictions. It is better to have *renvoi* rules that bite the bullet and forthrightly explain which part of a state's law they are willing to ignore and why. There is no way around the fact that choice of law always inherently means choosing something less than whole law.

### CHOICE OF LAW

A few simple examples illustrate how different choice-of-law rules avoid self-referential paradox. The simplest case is a state whose courts do no choice-of-law analysis at all. There are only zeroth-order substantive rules, and the courts apply them in any case they hear. This is what choice of law looks like in the pigheadedly parochial state of Pirelli, where there is only zeroth-order law, and which does not even acknowledge that there are other jurisdictions in the world.

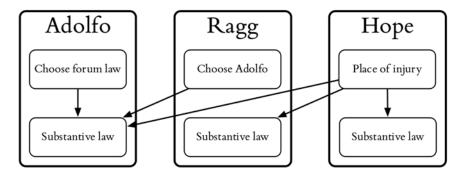
Pirelli

The next-simplest cases involve only first-order choice-of law. Here are some possible choice of law rules with only zeroth-order and first-order law.

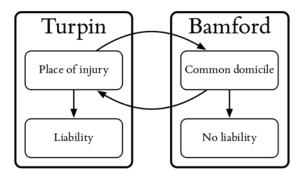
- The state of Adolfo, like the state of Pirelli, applies forum law in all cases. The difference is that it conducts an explicit (albeit kangaroo) choice-of-law inquiry before selecting its own substantive law.
- The state of Ragg takes a sarcastic suggestion of Brainerd Currie too seriously and applies Adolfo law in all cases, for predictability and ease of administration.

• The state of Hope follows the First Restatement place-of-the-injury rule, so it uses Hope law for batteries in Hope, Adolfo law for batteries in Adolfo, Ragg law for batteries in Ragg, and so on.

Adolfo, Ragg, and Hope all regard choice-of-law as a two-level hierarchy: the choice-of-law rules are first-order and they refer downwards to zeroth-order substantive rules, not sideways to each other.

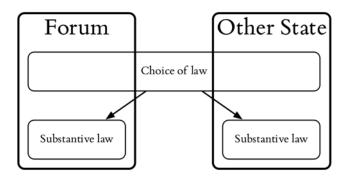


If first-order choice-of-law rules *do* refer sideways to each other, paradox quickly results. This is what goes wrong in the Turpin-Bamford standoff: choice of rules that attempt to defer to each other do not respect a proper type hierarchy.



This diagram should make even clearer how confused the idea of selecting another state's whole law is. Turpin only purports to select *Bamford*'s whole law. In what it regards as a domestic case, Turpin's first-order choice-of-law rules select Turpin's zeroth-order substantive law.

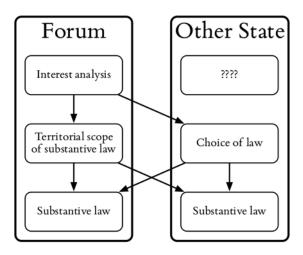
This perspective sheds light on how different choice-of-law theories avoid paradox. Take the First Restatement. One view of its theoretical commitments is that its rules never direct the forum to consider other states' choice-of-law rules because it is unwilling to entertain the possibility that they might be different from the forum's. Instead, choice of law is regarded as general law, on which all states agree, or should. This is either a fiction or a metaphysical assumption about the nature of law, but it coincidentally serves the purpose of preventing any possible cycles. If first-order choice-of-law rules are everywhere the same, there is no need for one state's to consider another's.



By way of contrast, consider interest analysis. One way of thinking about interest analysis is as legal interpretation: the forum court determines the territorial ambit of each state's substantive laws just as it would resolve any other question of legal scope. On this view, there are no distinctive choice-of-law issues. But this way of thinking skirts the edge of paradox, because choice of law *is* different: ordinary legal interpretation does not stare into the self-referential abyss. Some "modern" approaches, such as New York's Neumeier rules and David Cavers's principles of preference, are also two-level: they consider other states' substantive zeroth-order laws but not their first-order choice-of-law. The difference is that these approaches are forthrightly justified on policy grounds.

Another way of conceptualizing interest analysis is that a state's choice-of-law rules are *first-order* statements about the territorial scope of its *zeroth-order* substantive laws. They say that the state is interested in applying its substantive laws to certain cases, and not interested in applying them to others. But interest analysis itself is a *second-order* rule: it uses states' first-

order choice-of-law rules (including its own) to decide which states have an interest. It then uses second-order principles to pick one state's self-selecting first-order rule in cases when more than one state is interested. When a state does this, it is not really looking at the other state's entire law. Instead, it is looking to the other state's first-order choice-of-law rules to see whether the state claims an interest or not; it ignores the other state's second-order rules for selecting among interests. Put another way, interest analysis avoids paradox by limiting the types of interests that other states are allowed to have. <sup>14</sup>



Comparing this diagram with the First Restatement diagram shows that modern critics of traditional territorial jurisdiction-selecting rules have a point – but also that it is a point that can be turned around against them. States genuinely do have different policies about choice of law, so a First

<sup>&</sup>lt;sup>13</sup> In Roosevelt's terminology, the first-order rules are "rules of scope," and the second-order rules are "rules of priority." Roosevelt, *supra* note 12, at 1876-77. Notably, Brainerd Currie, the creator of interest analysis, argued that true conflicts should be resolved by choosing forum law. This is, as Roosevelt notes, a "rudimentary" rule of priority. *Id.* at 1877. William Baxter's comparative impairment approach to resolving true conflicts is an example of a more sophisticated second-order rule of priority.

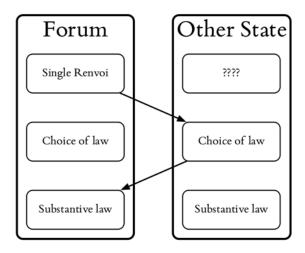
<sup>&</sup>lt;sup>14</sup> A critic of interest analysis might dispute my claim that it is a three-level system, and say instead that it is just another two-level system whose first-order rules uses the parties' domicile and naked forum preference to select a zeroth-order substantive law, but which dresses up the whole exercise in the rhetoric of interpretation.

Restatement-style first-order choice-of-law methodology that systematically ignores other states' first-order choice-of-law rules deliberately disregards an important aspect of other states' law. But so does interest analysis. It draws the line in a different place — between first-order and second-order choice-of-law rules — but it too draws a line to avoid paradox.

### THE RENVOI HIERARCHY

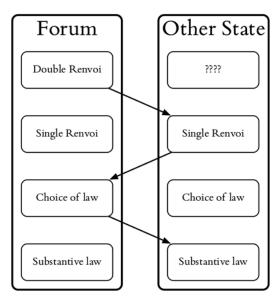
None of the cases so far involve *renvoi*. It cannot arise with a one- or two-level hierarchy. A state can take other states' choice of law into account without triggering a paradox only if there are three or more levels of law: zeroth-order substantive law, first-order choice-of-law rules, and second-order choice-of-law rules. It should be apparent that if there are exactly three levels, the second-order rules — we might as well call them *renvoi* rules — can safely refer only to first-order rules, not to each other.

The usual statement of "single *renvoi*," followed by some civil law systems, is that it will look to the law of another jurisdiction, but if that jurisdiction's choice of law returns the matter to the forum, the forum will "accept" the *renvoi* and apply its own domestic law.



Here, second-order *renvoi* rules do not refer to each other, only to first-order choice-of-law rules. Second-order *renvoi* rules avoid paradox by ignoring other states' second-order *renvoi* rules.

The hierarchy need not stop there. Renvoi rules can non-paradoxically refer to other jurisdictions' renvoi rules – as long as they refer only to some lower level of renvoi rules. "Double renvoi," purportedly followed by some jurisdictions, breaks the cycle not by always accepting or rejecting the renvoi, but by whether the other jurisdiction would accept or reject the renvoi. This is a fourlevel hierarchy:



Double *renvoi* is sometimes described as selecting an appropriate jurisdiction and then applying whatever law that jurisdiction's courts would, including its attitude toward the *renvoi*. But this is not quite right. If the other jurisdiction is also a double-*renvoi* jurisdiction, treating it like a single-*renvoi* jurisdiction ignores the fourth level.

Double *renvoi* is properly named: it stops after at most two applications of *renvoi*. One could imagine triple *renvoi*, quadruple *renvoi*, and even higher-order *renvoi*, just like the infinite hierarchy of set types in the *Principia*. They are all logically possible, although they serve increasingly infinitesimal purposes. But there is no infinite *renvoi*. The cycle must be broken. <sup>16</sup>

 $Mrs.\ Lovett\ (Wearily):\ Just\ how\ many\ bells\ are\ there?$ 

Beadle: Twelve.

Steven Sondheim, Finishing the Hat 371 (2010).

<sup>&</sup>lt;sup>15</sup> Or:

<sup>&</sup>lt;sup>16</sup> Even saying "take the limit of the infinite series . . ." does not help. For one thing, not all infinite series converge in the necessary sense. For another, computing the outcomes of infinite processes is often provably impossible. For a third, what if Turpin and Bamford both try to take the limit and then prescribe opposite results based on what they find? The paradoxical cycle simply recreates itself one layer of abstraction higher.

That is what *renvoi* is: cycle-breaking. Mathematics cannot tell law how to break the cycle: that is a question of policy, which pure mathematics is singularly unfit to address. But it can remind lawyers what they are doing when they pick a *renvoi* rule. They are breaking cycles of self-reference by imposing a hierarchy on choice-of-law rules in which some of those rules necessarily ignore others. Any attempt to avoid that fact is like trying to find the barber who shaves all the men who do not shave themselves: doomed to end badly, in madness and noise.

