

# Quantifying Copyright

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# I. Information Theory

# Three key ideas

- Digital encoding
- Counting bits
- Compression

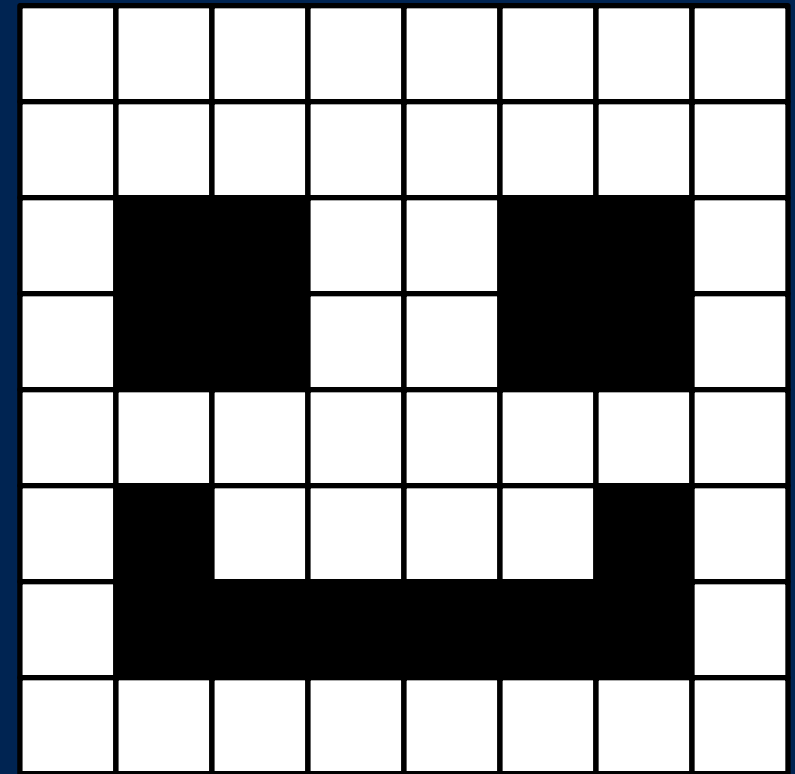
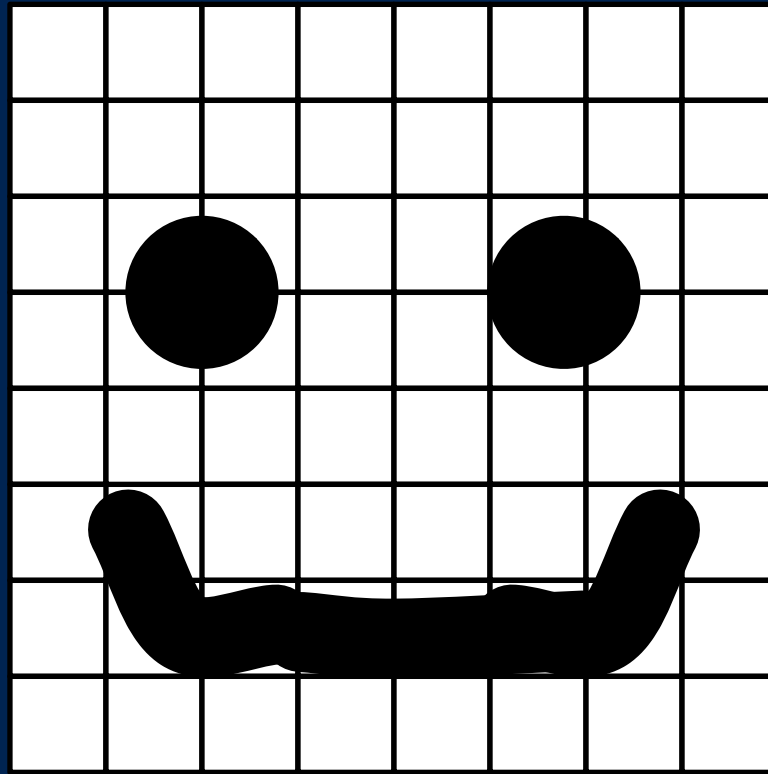
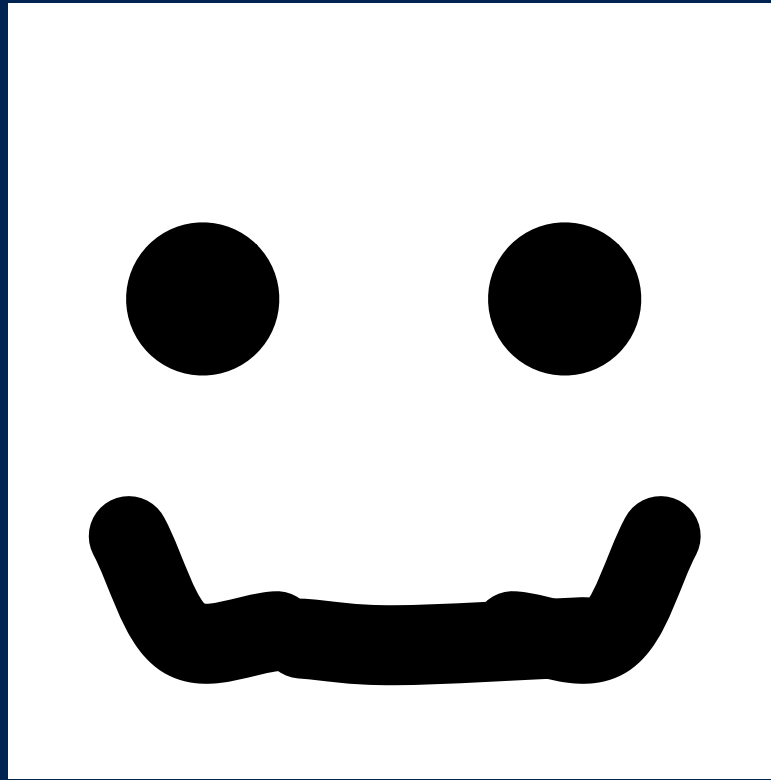
# 1. Digital encoding

H e l l o !

72 101 108 108 111 33

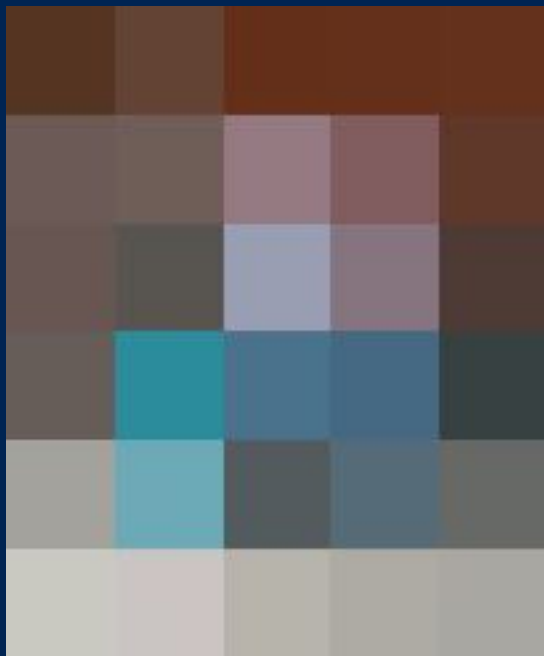
01001000 01100101 01101100 01101100 01101111 00100001

# 1. Digital encoding

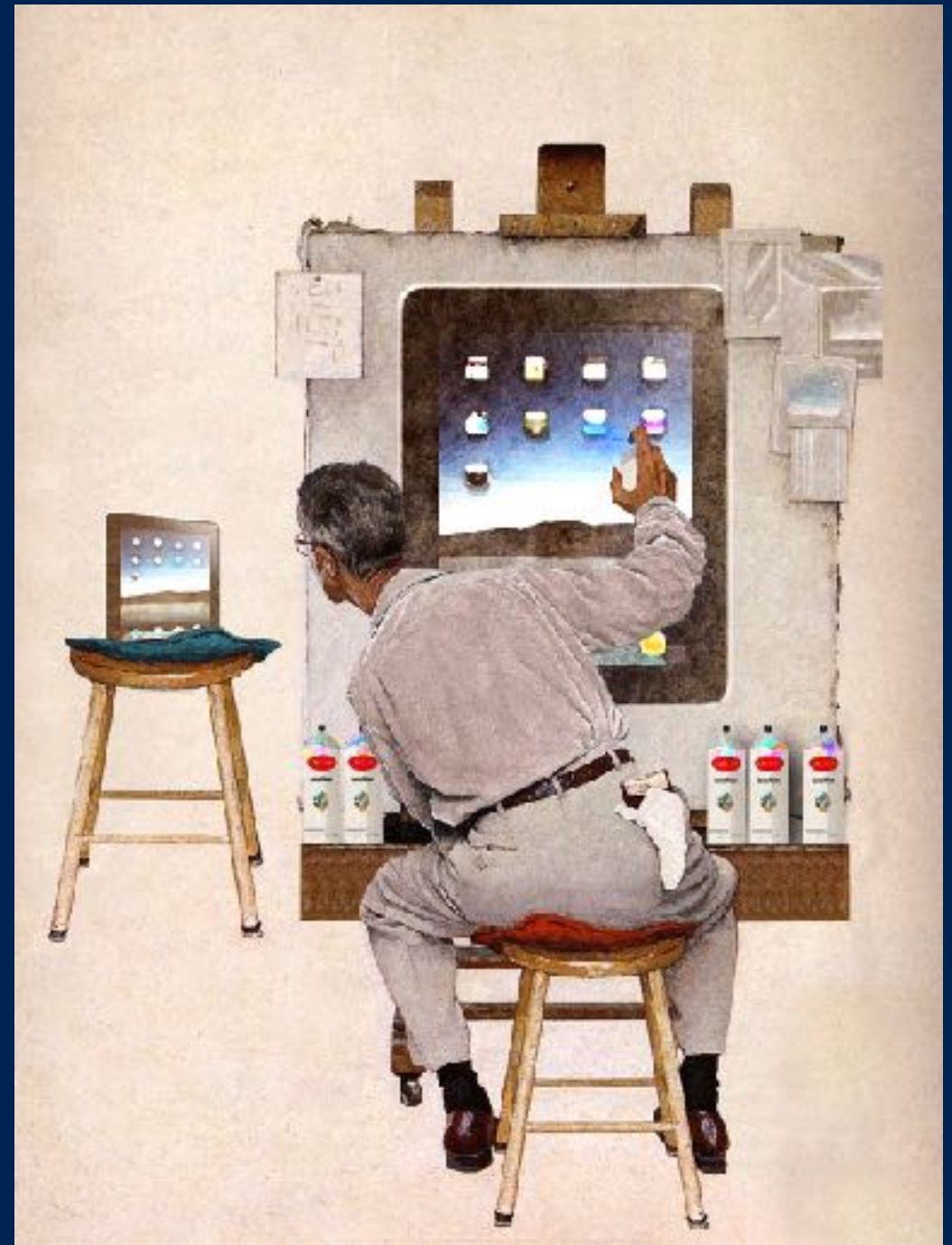
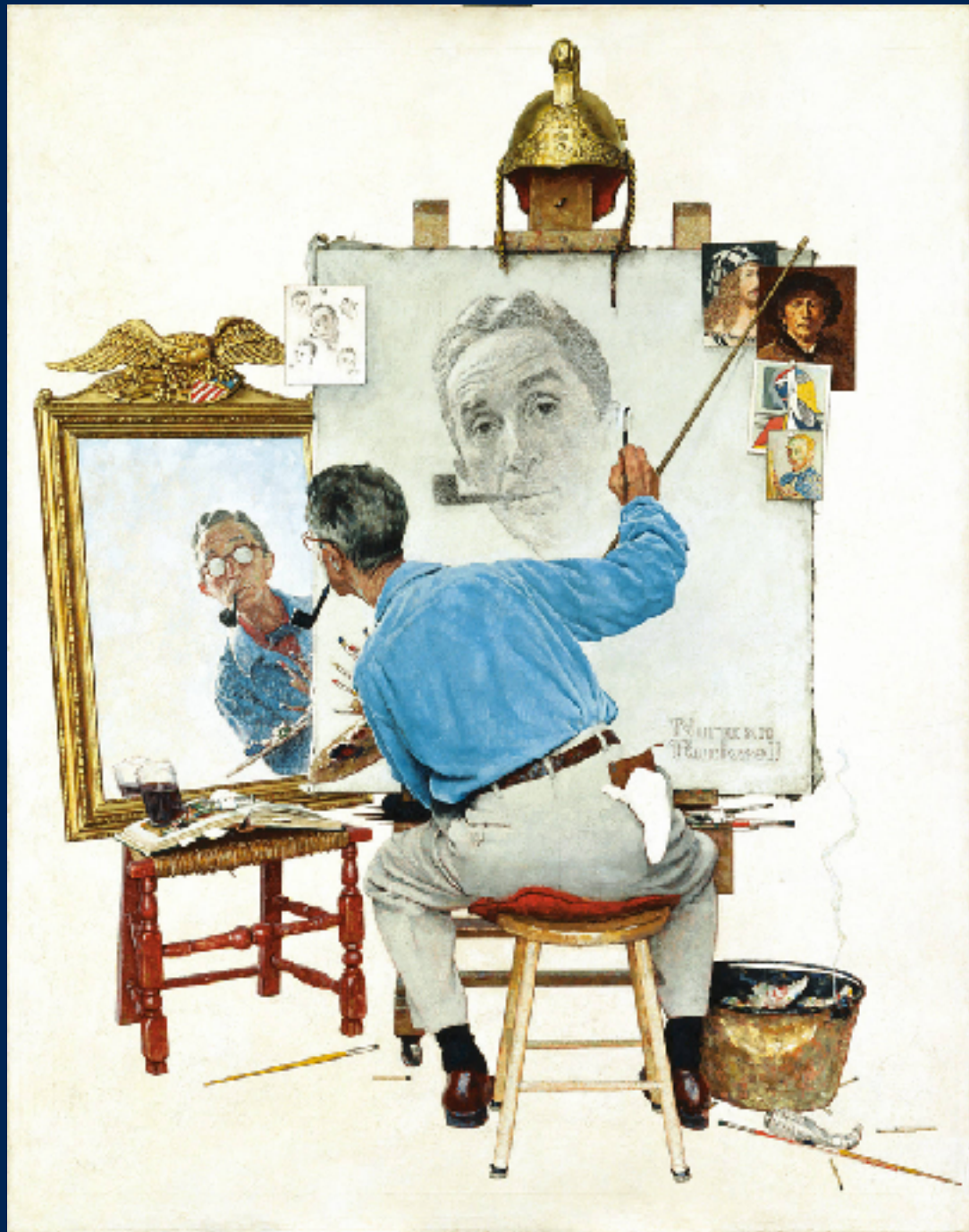


1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	0	0	1	1	0	0	1
1	0	0	1	1	0	0	1
1	1	1	1	1	1	1	1
1	0	1	1	1	1	0	1
1	0	0	0	0	0	0	1
1	1	1	1	1	1	1	1

# 1. Digital encoding



# 1. Digital encoding



## 2. Counting bits

You Got the Right One, Uh-Huh

232 bits



## 2. Counting bits

Thou still unravished bride of quietness,  
Thou foster-child of silence and slow time,  
Sylvan historian, who canst thus express  
A flowery tale more sweetly than our rhyme

...

“Beauty is truth, truth beauty,—that is all  
Ye know on earth, and all ye need to know”

17,544 bits

# 3. Compression

English	UTF-8	WTF-8
A	01000001	01000000100000000000000000000000 00000000000000000000000000000000
B	01000010	01000010000000000000000000000000 00000000000000000000000000000000
C	01000011	01000011000000000000000000000000 00000000000000000000000000000000
D	01000100	01000100000000000000000000000000 00000000000000000000000000000000

# 3. Compression

Happy\_birthday\_to\_you←  
Happy\_birthday\_to\_you←  
Happy\_birthday\_dear\_X←  
Happy\_birthday\_to\_you←

704 bits

# 3. Compression

1:Happy\_birthday

2:\*1\_to\_you←

\*2\*2\*1\_dear\_X\*2

368 bits

# II. Kolmogorov Complexity

# Two branches of information theory

- (Shannon) information theory:  
communications, processes, encodings, noise
  - Properties of *communication systems*
  - Cf. Fromer, Scafidi
- (Algorithmic) information theory:  
computation, compressibility, encodings
  - Properties of *individual texts*

The Kolmogorov complexity ( $K$ ) of a work ( $x$ ) is the shortest (min) length ( $l$ ) of a program ( $p$ ) which produces ( $\rightarrow$ ) the work

$$K(x) = \min_{p \rightarrow x} l(p)$$

*A work is only as complex as its shortest encoding*

# Naive encodings

XXXXXXXXXX (10 Xs)	80 bits
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XXXXXXXXX...XXXXXXX (100 Xs)	800 bits
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XXXXXXXXX...XXXXXXXXXX (1000 Xs)	8000 bits
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XXXXXXXXX...XXXXXXXXXXXXX (10000 Xs)	80000 bits
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# Sophisticated encodings

XXXXXXXXXX (10 Xs)	80 bits
--------------------	---------

for(\$i=0;\$i<10;\$i++){print"X"}	240 bits
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XXXXXXXXX...XXXXXXXX (100 Xs)	800 bits
-------------------------------	----------

for(\$i=0;\$i<100;\$i++){print"X"}	248 bits
------------------------------------	----------

XXXXXXXXX...XXXXXXXXXX (1000 Xs)	8000 bits
----------------------------------	-----------

for(\$i=0;\$i<1000;\$i++){print"X"}	256 bits
-------------------------------------	----------

XXXXXXXXX...XXXXXXXXXXXX (10000 Xs)	80000 bits
-------------------------------------	------------

for(\$i=0;\$i<10000;\$i++){print"X"}	264 bits
--------------------------------------	----------

# Comparing works



Kolmogorov complexity

$$K(x) = \min_{p \rightarrow x} l(p)$$

Conditional Kolmogorov complexity

$$K(x | \gamma) = \min_{p(\gamma) \rightarrow x} l(p)$$

$K(x)$  = complexity of  $x$

$K(x|\gamma)$  = complexity of  $x$  not due to  $\gamma$

$K(x) - K(x|\gamma)$  = complexity of  $x$  due to  $\gamma$



$$K(\text{Mona Lisa}) = \text{high}$$

$$K(\text{Mona Lisa} \mid \text{Mona Lisa}) = \text{medium}$$

$$K(\text{Mona Lisa}) - K(\text{Mona Lisa} \mid \text{Mona Lisa}) = \text{high}$$

$$K(\text{Mona Lisa} \mid \text{Sunflowers}) = \text{high}$$

$$K(\text{Mona Lisa}) - K(\text{Mona Lisa} \mid \text{Sunflowers}) = \text{low}$$

# III. Copyright

# Tentative idea #1: counting arguments

- *Feist*: there are  $(250,000,000!)/(50,000!)$   
 $(249,950,000!)$  ways to select 50,000 listings
- $\sim 1,500,000$  bits to describe an arbitrary selection
- There are  $50,000!$  ways to arrange those listings
  - $\sim 750,000$  bits to describe an arbitrary ordering
- But the *actual* selection and arrangement and require far fewer bits: too few for a copyright

# Tentative idea #2: quantify factor three

- *Perfect 10*: how much of a 1000x1000 image does a 100x100 thumbnail copy?
  - 100%, because it's the “whole” image?
  - 1%, because it has 1/100 as many pixels?
- Why not compare .JPG file sizes?



# Other tentative ideas

- Rule-based creativity: can't extract more bits of expression than you put in
- Merger kicks in when there are “only a limited number of ways” to express an idea
- Scènes à faire are about predictability: in a hard-boiled detective novel, it adds almost no new information to learn that the hero drinks

# More ambitious intuition

- Use  $K(x)$  to measure expression
- Use  $K(x|\gamma)$  to perform filtration
- Use  $K(x|\gamma)$  to measure similarity

# Objections

- *Objection:  $K$  is uncomputable*
- *Objection:  $K$  ignores psychology and aesthetics*

# Ways to make progress?

- Input into expert testimony in cases involving technical subject matter (e.g., software)?
- Lossy compression, psychology, and aesthetics?

Questions?





$K(x|\gamma)$  = complexity of  $x$  not due to  $\gamma$

$K(x) - K(x|\gamma)$  = complexity of  $x$  due to  $\gamma$

$K(x|\gamma, z)$  = complexity of  $x$  not due to  $\gamma$  or  $z$

$K(x|\gamma) - K(x|\gamma, z)$  = complexity of  $x$  due to  $z$  but not  $\gamma$